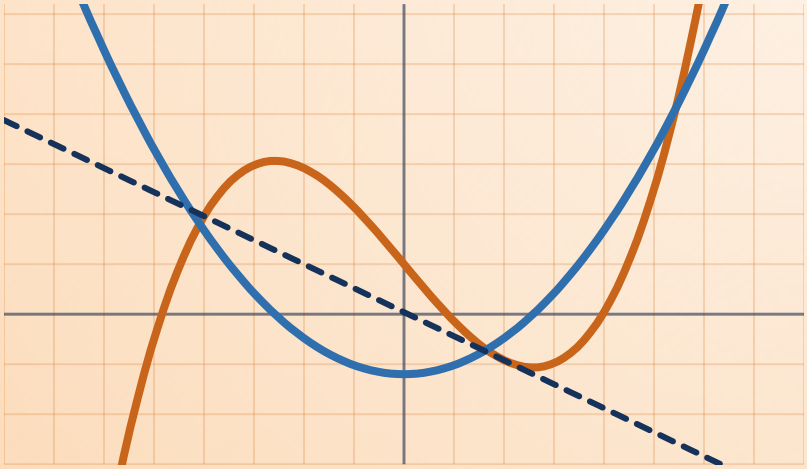
KECUDO

2026 EDITION · COLLEGE MATH SERIES



CALCULUS I

Class B2

The Derivative & Differentiation Rules

The mathematics of instant change

PREMIUM EDITION · KECUDO · B2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Tangent Lines & Rates of Change

Class B1 showed that the slope of a tangent line is the limit of secant slopes. That limit is so important it gets its own name: the **derivative**. First, let's compute a few tangents by hand.

WORKED EXAMPLE **Tangent line to $f(x) = x^2$ at $x = 3$**

1. Slope: $\lim_{h \to 0} [(3 + h)^2 - 9]/h = \lim_{h \to 0} (6 + h) = 6$.
2. Point: (3, 9). Point-slope form: $y - 9 = 6(x - 3)$.

✓ $y = 6x - 9$

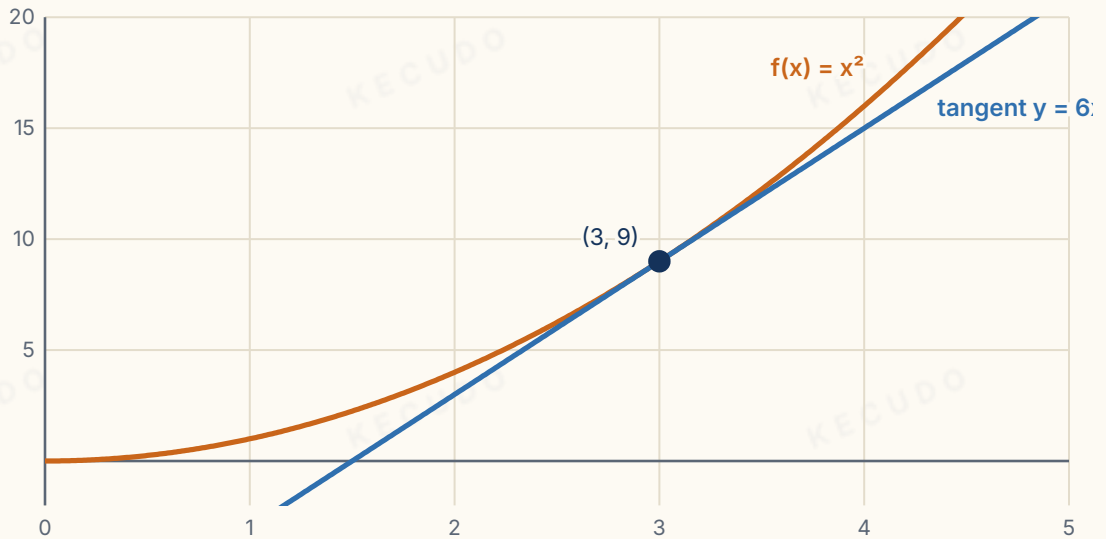


FIG 1.1 The tangent line touches the parabola at (3, 9) and has the same steepness there.

If f measures...	the rate of change f' measures...	Units
position (m) vs time (s)	velocity	m/s
cost ($) vs items made	marginal cost	$ per item
members vs months	growth rate	members per month
volume (L) vs time (min)	flow rate	L/min

WORKED EXAMPLE **The Kecudo e-bike at $t = 2$**

1. $\lim_{h \to 0} [5(2 + h)^2 - 20]/h = \lim_{h \to 0} (20 + 5h)$.

✓ **20 m/s**

KEY IDEA

Slope of the tangent = instantaneous rate of change. Same number, two meanings: one geometric, one physical.

FOUNDATION

Chapter 2

Definition of the Derivative

$$f'(x) = \lim_{h \to 0} [f(x + h) - f(x)]/h$$

This is a **function**: plug in any x and it returns the slope there. Other notations: dy/dx, $d/dx\ f(x)$, y'.

WORKED EXAMPLE **Find $f'(x)$ for $f(x) = x^2 + 3x$**

1. $f(x + h) - f(x) = 2xh + h^2 + 3h$.
2. Divide by h: $2x + h + 3$. Let $h \to 0$.

✓ $f'(x) = 2x + 3$

WORKED EXAMPLE **Find $f'(x)$ for $f(x) = 1/x$**

1. $[1/(x + h) - 1/x]/h = -h/[x(x + h)h] = -1/[x(x + h)]$.

✓ $f'(x) = -1/x^2$

WORKED EXAMPLE **Find $f'(x)$ for $f(x) = \sqrt{(x)}$**

1. Rationalize: $[\sqrt{(x + h)} - \sqrt{(x)}]/h = 1/[\sqrt{(x + h)} + \sqrt{(x)}]$.

✓ $f'(x) = 1/(2\sqrt{(x)})$

CHECK YOUR UNDERSTANDING

Chapter 1: Tangent Lines & Rates of Change

5 questions · your answer locks when you choose it

1. What is the slope of the tangent to $f(x) = x^2$ at $x = 3$?

- **A** 12
- **B** 9
- **C** 3
- **D** 6

2. What is the tangent line to $f(x) = x^2$ at (3, 9)?

- **A** $y = 6x + 9$
- **B** $y = 3x$
- **C** $y = 6x - 9$
- **D** $y = 9x - 18$

3. If f is position in meters and t is in seconds, the units of the rate of change are:

- **A** m
- **B** m/s
- **C** m²
- **D** s/m

4. For $s(t) = 5t^2$, what is the velocity at $t = 2$?

- **A** 25 m/s
- **B** 10 m/s
- **C** 40 m/s
- **D** 20 m/s

5. The slope of a tangent line equals:

- **A** the instantaneous rate of change
- **B** the average rate over [0, 1]
- **C** the y-intercept
- **D** the area under the curve

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. The derivative $f'(a)$ gives:

- **A** the average of f
- **B** the value $f(a)$
- **C** the slope of the tangent at a
- **D** the area under f

2. Which notation does NOT mean the derivative of $y = f(x)$?

- **A** dy/dx
- **B** $\int f(x)\,dx$
- **C** $f'(x)$
- **D** y'

3. Which function is not differentiable at $x = 0$?

- **A** x^2
- **B** e^x
- **C** $\cos x$
- **D** $|x|$

4. What is $d/dx\,[e^x]$?

- **A** 1
- **B** e^x
- **C** e
- **D** $x\,e^{x-1}$

5. The Chain Rule is for:

- **A** a function inside another function
- **B** quotients
- **C** constants
- **D** products

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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class B3 · Applications of Derivatives