KECUDO

2026 EDITION · COLLEGE MATH SERIES



CALCULUS II

Class C2

Applications of Integration

Areas, volumes, work and more

PREMIUM EDITION · KECUDO · C2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Area Between Curves

$$A = \int_a^b [f(x) - g(x)]\,dx, \text{ where } f \geq g \text{ on } [a, b]$$

WORKED EXAMPLE **Area between $y = 2x$ and $y = x^2$**

1. They meet at $x = 0$ and $x = 2$; $2x$ is on top.
2. $\int_0^2 (2x - x^2)\,dx = 4 - 8/3$.

✓ **4/3**

Integrating with respect to y

When curves are written as $x = \ldots$, use horizontal strips: $A = \int_c^d [\text{right} - \text{left}]\,dy$.

WORKED EXAMPLE **Region between $x = y^2$ and $x = y + 2$**

1. They meet where $y^2 = y + 2$: $y = -1$ and $y = 2$.
2. $\int_{-1}^2 (y + 2 - y^2)\,dy = 10/3 + 7/6$.

✓ **9/2**

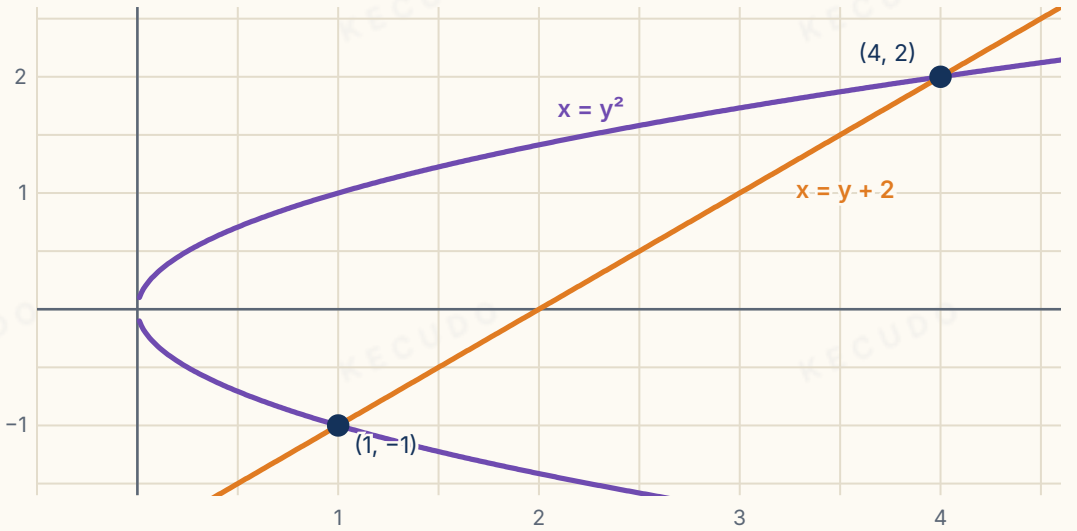


FIG 1.1 Here horizontal strips are easier: one formula for right minus left.

WORKED EXAMPLE **When curves cross: $\sin x$ and $\cos x$ on $[0, \pi/2]$**

1. They cross at $\pi/4$; split there.
2. By symmetry: $2\int_0^{\pi/4} (\cos x - \sin x)\, dx = 2(\sqrt{(2)} - 1)$.

✓ ≈ 0.828

FOUNDATION

Chapter 2

Volume by Slicing

$V = \int_a^b A(x)\,dx$, where $A(x)$ is the area of the cross-section at x

WORKED EXAMPLE **A pyramid with a square base of side b and height h**

1. At height y from the top, the side is $b(y/h)$, so $A = b^2\,y^2/h^2$.
2. $\int_0^h b^2\,y^2/h^2\,dy = b^2\,h/3$.

✓ **$V = b^2\,h/3$: a third of the box**

WORKED EXAMPLE **A solid whose base is the disk $x^2 + y^2 \le 4$, with square cross-sections perpendicular to the x-axis**

1. The side is $2\sqrt{(4 - x^2)}$, so $A(x) = 4(4 - x^2)$.
2. $\int_{-2}^{2} (16 - 4x^2)\,dx = 64 - 64/3$.

✓ **128/3**

CAVALIERI'S PRINCIPLE

Two solids with the same cross-sectional areas at every height have the same volume, however they are shaped.

CHECK YOUR UNDERSTANDING

Chapter 1: Area Between Curves

5 questions · your answer locks when you choose it

1. What is the area between $y = x$ and $y = x^2$ on [0, 1]?

- **A** 1/2
- **B** 5/6
- **C** 1/3
- **D** 1/6

2. For curves of the form $x = g(y)$, integrate:

- **A** top minus bottom, with respect to x
- **B** right minus left, with respect to y
- **C** left minus right
- **D** with polar coordinates

3. What is the area between $y = 4$ and $y = x^2$?

- **A** 32/3
- **B** 16/3
- **C** 8
- **D** 16

4. If the curves cross inside $[a, b]$:

- **A** split at the crossing
- **B** ignore it
- **C** use absolute value of the final answer
- **D** double the integral

5. What is the area between $y = x^3$ and $y = x$ on [0, 1]?

- **A** 1/2
- **B** 1/4
- **C** 3/4
- **D** 1/12

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. The area between two curves integrates:

A top plus bottom
B the larger function only
C top minus bottom
D top times bottom

2. The disk method about the x-axis integrates:

A $2\pi x\, f(x)$
B $\pi[f(x)]^2$
C $2\pi f(x)$
D $f(x)$

3. Work equals:

A mass times speed
B the integral of force over distance
C force divided by distance
D pressure times area

4. A probability density function must have total area:

A any positive value
B 100
C 1
D 0

5. A separable equation can be written as:

A $dy/dx = g(x) + h(y)$
B $dy/dx = 0$
C $y'' = 0$
D $dy/dx = g(x)h(y)$



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class C3 · Sequences & Infinite Series