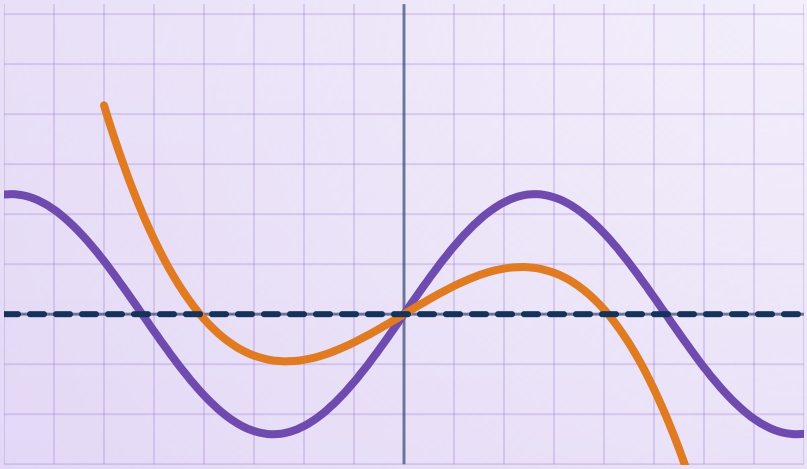
KECUDO

2026 EDITION · COLLEGE MATH SERIES



CALCULUS II

Class C4

Parametric, Polar & Power Series

New coordinates and infinite polynomials

PREMIUM EDITION · KECUDO · C4

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Parametric Curves

A **parametric curve** gives both coordinates as functions of a third variable, usually time: $x = f(t)$, $y = g(t)$. It records not just the shape but the motion along it.

WORKED EXAMPLE **Circle and projectile**

1. $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$: the unit circle, traced counterclockwise.
2. A ball launched at 20 m/s in each direction: $x = 20t$, $y = 20t - 4.9t^2$. It lands when $y = 0$: $t \approx 4.08$ s, $x \approx 81.6$ m.

✓ **Eliminate t to find the shape; keep t to know the timing**

WORKED EXAMPLE **The valve on a Kecudo wheel (radius $r = 0.35$ m) rolling along the ground**

1. After turning through angle t, the wheel has rolled rt forward.
2. Valve position: $x = r(t - \sin t)$, $y = r(1 - \cos t)$.

✓ **This curve is a cycloid**

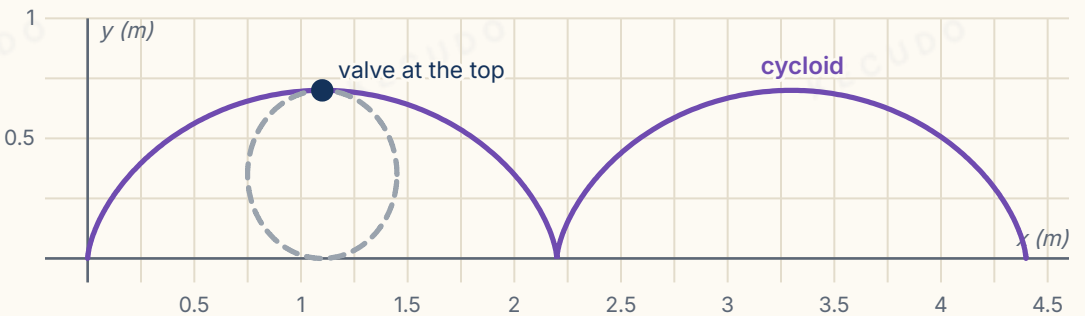


FIG 1.1 Two turns of a 0.35 m wheel; each arch is $2\pi(0.35) \approx 2.2$ m long along the ground.

ELIMINATING THE PARAMETER

Solve one equation for t and substitute: $x = 2t + 1$, $y = t^2$ gives $y = ((x - 1)/2)^2$, a parabola.

FOUNDATION

Chapter 2

Calculus with Parametric Curves

$$dy/dx = (dy/dt)/(dx/dt) \text{ and } L = \int_a^b \sqrt{((dx/dt)^2 + (dy/dt)^2)}\, dt$$

WORKED EXAMPLE **Slope of the unit circle $x = \cos t$, $y = \sin t$ at $t = \pi/4$**

1. $dy/dx = \cos t/(-\sin t) = -\cot t$.

✓ **-1 at $t = \pi/4$**

WORKED EXAMPLE **Length of one arch of the Kecudo cycloid**

1. $(dx/dt)^2 + (dy/dt)^2 = r^2[(1 - \cos t)^2 + \sin^2 t] = 2r^2(1 - \cos t) = 4r^2 \sin^2(t/2)$.
2. $L = \int_0^{2\pi} 2r \sin(t/2)\, dt = 8r$.

✓ **8(0.35) = 2.8 m per turn, while the bike moves only 2.2 m**

WORKED EXAMPLE **Speed of the valve**

1. Speed $= \sqrt{((dx/dt)^2 + (dy/dt)^2)} \cdot \omega = 2r\,\omega\,|\sin(t/2)|$.

✓ **Zero at the bottom, twice the bike's speed at the top**

HORIZONTAL AND VERTICAL TANGENTS

Horizontal where $dy/dt = 0$ (and $dx/dt \neq 0$); vertical where $dx/dt = 0$ (and $dy/dt \neq 0$).

CHECK YOUR UNDERSTANDING

Chapter 1: Parametric Curves

5 questions · your answer locks when you choose it

1. $x = 3\cos t$, $y = 3\sin t$ traces:

- **A** a parabola
- **B** an ellipse with axes 3 and 1
- **C** a circle of radius 3
- **D** a line

2. Eliminate t: $x = t + 1$, $y = 2t$.

- **A** $y = 2x + 2$
- **B** $y = x/2$
- **C** $y = 2x$
- **D** $y = 2x - 2$

3. For $x = 20t$, $y = 20t - 4.9t^2$, when does the ball land?

- **A** ≈ 2.04 s
- **B** ≈ 4.08 s
- **C** 20 s
- **D** ≈ 8.16 s

4. The path of a point on a rolling wheel is called a:

- **A** cycloid
- **B** spiral
- **C** hyperbola
- **D** catenary

5. What extra information does a parametric curve carry?

- **A** When and in which direction the point moves
- **B** Nothing extra
- **C** Only the slope
- **D** Only the area

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. A parametric curve gives:

A r as a function of θ | **B** y as a function of x only

C x and y as functions of a parameter | **D** a single number

2. In polar coordinates, $x =$

A $r \sin \theta$ | **B** $\theta \cos r$

C r/θ | **D** $r \cos \theta$

3. A power series converges:

A everywhere always | **B** only at integers

C on an interval around its center | **D** nowhere

4. The Maclaurin series of e^x is:

A $\Sigma\, n\, x^n$ | **B** $\Sigma\, x^n/n!$

C $\Sigma\, x^n$ | **D** $\Sigma\, (-1)^n\, x^{2n+1}/(2n+1)!$

5. Euler's formula states $e^{i\theta} =$

A $i\,\theta$ | **B** $\cos \theta$

C $\cos \theta + i \sin \theta$ | **D** $\sin \theta + i \cos \theta$



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class D1 · Vectors & 3D Geometry (Calculus III)