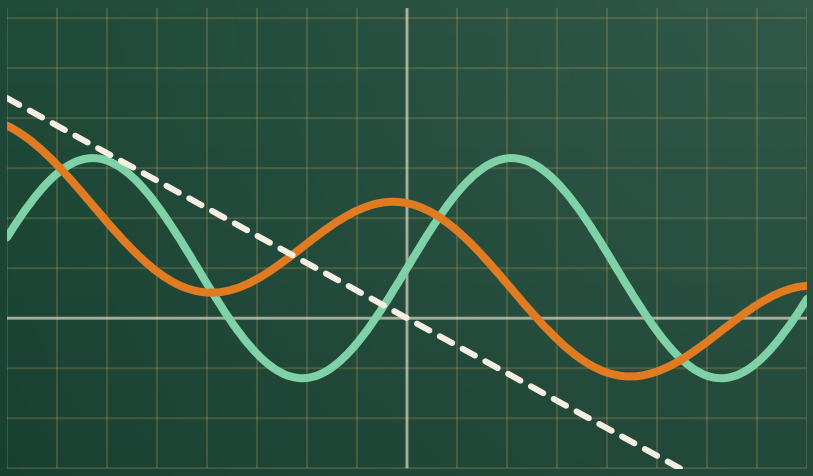
KECUDO

2026 EDITION · COLLEGE MATH SERIES



CALCULUS III

Class D1

Vectors & 3D Geometry

Arrows, lines, planes and curves in space

PREMIUM EDITION · KECUDO · D1

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

3D Coordinates

Space needs three numbers. A point (x, y, z) sits x units along the x-axis, y along the y-axis and z up. The axes follow the **right-hand rule**: curl your fingers from x to y and your thumb points along z.

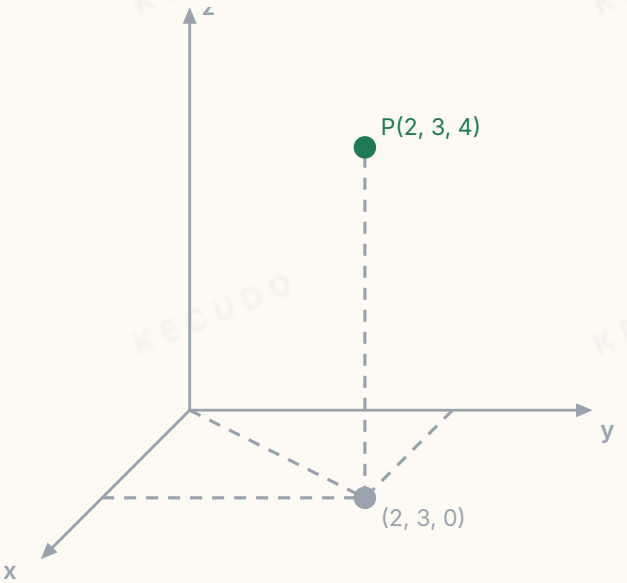


FIG 1.1 To reach $P(2, 3, 4)$: walk 2 along x, 3 along y, then climb 4.

$$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)}$$

WORKED EXAMPLE **Distance from the shop roof (1, 2, 3) to a drone at (4, 6, 15)**

1. $\sqrt{(3^2 + 4^2 + 12^2)} = \sqrt{(169)}$.

✓ **13 units**

WORKED EXAMPLE **Find the center and radius of $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$**

1. Complete the square: $(x - 1)^2 + (y + 2)^2 + (z - 3)^2 = 2 + 1 + 4 + 9 = 16$.

✓ **Sphere: center $(1, -2, 3)$, radius 4**

EQUATIONS IN SPACE

One equation usually makes a surface: $z = 0$ is the xy-plane, not a line. You need two equations to pin down a curve.

FOUNDATION

Chapter 2

Vectors

A **vector** has length and direction but no fixed position. In components, $v = \langle v_1, v_2, v_3 \rangle$. Vectors add tip-to-tail and scale by numbers.

$$|v| = \sqrt{(v_1^2 + v_2^2 + v_3^2)} \text{ and unit vector } u = v/|v|$$

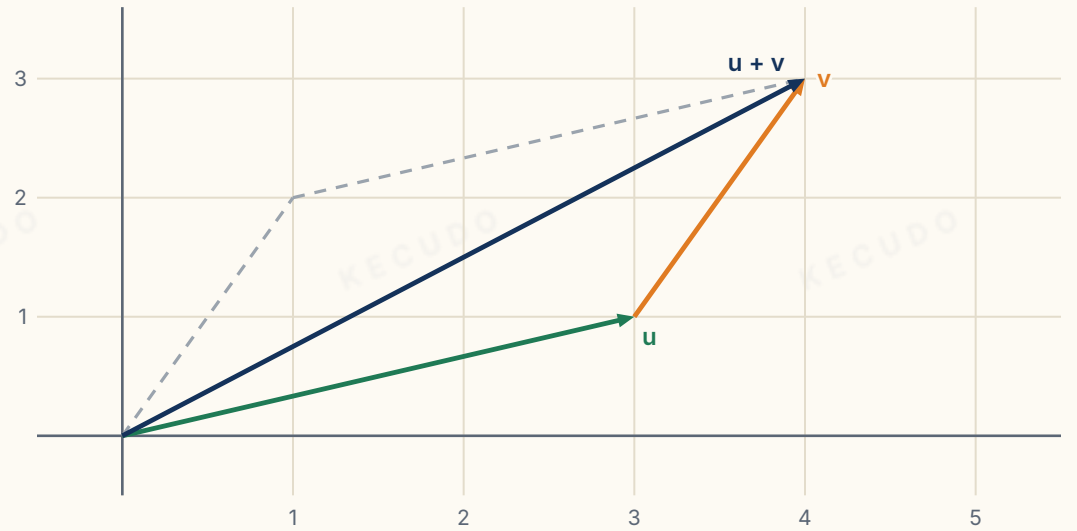


FIG 2.1 Tip-to-tail: $u + v$ is the diagonal of the parallelogram.

WORKED EXAMPLE **The drone flies from the shop (0, 0, 0) to a rider at (300, 400, 120) m**

1. Displacement $\langle 300, 400, 120 \rangle$; length $\sqrt{(264{,}400)} \approx 514.2$ m.
2. Unit direction $\approx \langle 0.583, 0.778, 0.233 \rangle$.

✓ **514 m straight-line flight**

CHECK YOUR UNDERSTANDING

Chapter 1: 3D Coordinates

5 questions · your answer locks when you choose it

1. What is the distance from $(0, 0, 0)$ to $(2, 3, 6)$?

A 49 | **B** 11

C 7 | **D** $\sqrt{(11)}$

2. In 3D, the equation $y = 2$ describes:

A a plane | **B** a sphere

C a line | **D** a point

3. What are the center and radius of $(x - 1)^2 + y^2 + (z + 2)^2 = 9$?

A $(1, 0, 2)$, 3 | **B** $(1, 0, -2)$, 3

C $(-1, 0, 2)$, 3 | **D** $(1, 0, -2)$, 9

4. What is the midpoint of $(2, 0, 4)$ and $(6, 2, -2)$?

A $(4, 1, 1)$ | **B** $(2, 1, 3)$

C $(8, 2, 2)$ | **D** $(4, 2, 1)$

5. Which point lies on the z-axis?

A $(0, 0, 5)$ | **B** $(0, 5, 0)$

C $(5, 5, 0)$ | **D** $(5, 0, 0)$

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. The dot product $a \cdot b$ is:

A a number
B a plane
C a vector
D a matrix

2. The cross product $a \times b$ is:

A a number
B a vector perpendicular to a and b
C parallel to a
D always zero

3. A plane is determined by a point and:

A a radius
B a curvature
C a normal vector
D a direction vector only

4. The derivative $r'(t)$ of a vector function is:

A the normal plane
B the curvature
C the position
D the tangent (velocity) vector

5. In spherical coordinates, ρ is:

A the height
B the angle from x
C the distance from the origin
D the distance from the z-axis



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class D2 · Partial Derivatives & Optimization