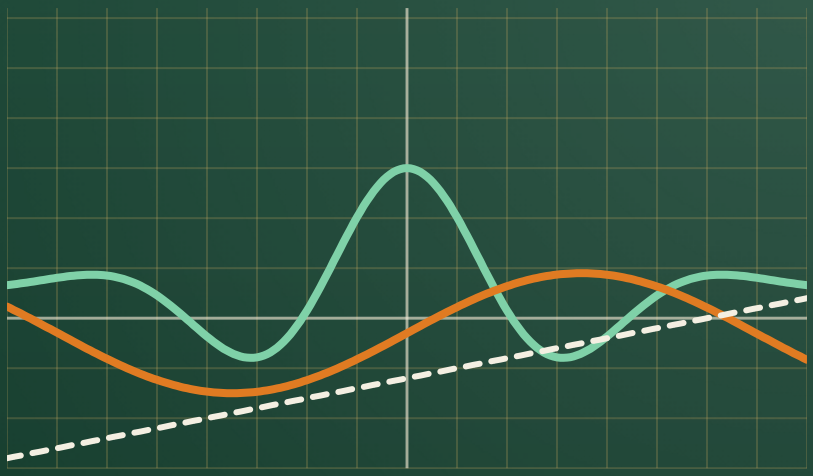
KECUDO

2026 EDITION · COLLEGE MATH SERIES



CALCULUS III

Class D4

Vector Calculus

Fields, flow, circulation and flux

PREMIUM EDITION · KECUDO · D4

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Vector Fields

A **vector field** attaches a vector to every point: $F(x, y) = \langle P(x, y), Q(x, y) \rangle$ in the plane or $\langle P, Q, R \rangle$ in space. Wind maps, water currents, gravity and electric fields are all vector fields.

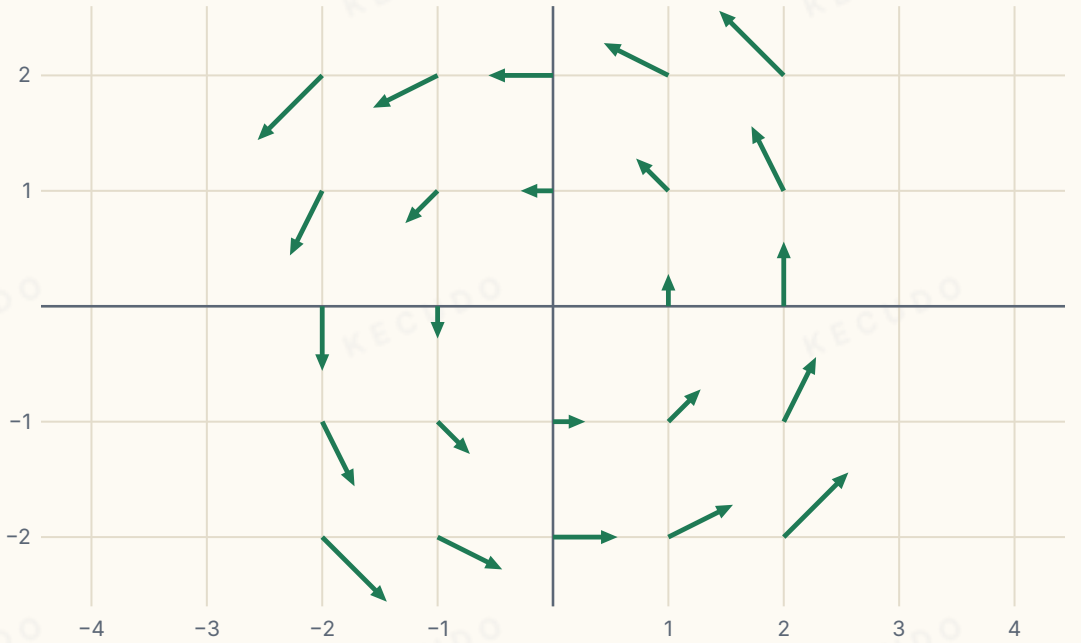


FIG 1.1 The rotation field $F = \langle -y, x \rangle$: arrows circle the origin and grow longer farther out.

WORKED EXAMPLE **Three classic fields**

1. $F = \langle -y, x \rangle$: counterclockwise swirl (like water around a drain).
2. $F = \langle x, y \rangle$: pointing straight out from the origin (a source).
3. $F = -\mathrm{GMm}\, r/|r|^3$: gravity, always pointing toward the mass.

✓ **Picture them by drawing a few arrows**

GRADIENT FIELDS

Every function f gives a field ∇f. Such fields are called **conservative**, and f is a **potential**. Example: $f = x^2 y$ gives $\nabla f = \langle 2xy, x^2 \rangle$.

WORKED EXAMPLE **Wind over the Kecudo loop**

1. Measured wind $W(x, y) = \langle 3 + 0.1y, -0.2x \rangle$ m/s (with x and y in km).
2. At $(5, 0)$: $W = \langle 3, -1 \rangle$, speed $\sqrt{(10)} \approx 3.2$ m/s.

✓ **A field is a map of arrows**

FOUNDATION

Chapter 2

Line Integrals

$$\int_C f\,ds = \int_a^b f(r(t))\,|r'(t)|\,dt \text{ and } \int_C F\cdot dr = \int_a^b F(r(t))\cdot r'(t)\,dt$$

The first adds up a quantity along a curve (mass of a wire, for example). The second adds up the component of a field along the motion: the **work** done by F.

WORKED EXAMPLE **Mass of a circular wire $x^2 + y^2 = 1$ with density $f = x^2 + y^2$**

1. $f = 1$ on the circle, $|r'| = 1$.
2. $\int_0^{2\pi} 1\,dt$.

✓ **2π**

WORKED EXAMPLE **Work by $F = \langle x, y\rangle$ along the segment from (0, 0) to (1, 2)**

1. $r(t) = \langle t, 2t\rangle$, $r' = \langle 1, 2\rangle$, $F \cdot r' = t + 4t = 5t$.
2. $\int_0^1 5t\,dt$.

✓ **5/2**

WORKED EXAMPLE **The swirl $F = \langle -y, x\rangle$ around the circle of radius 2, counterclockwise**

1. $r = \langle 2\cos t, 2\sin t\rangle$, $F \cdot r' = 4\sin^2 t + 4\cos^2 t = 4$.
2. $\int_0^{2\pi} 4\,dt = 8\pi$.

✓ **Positive: the field pushes along the motion all the way round**

CHECK YOUR UNDERSTANDING

Chapter 1: Vector Fields

5 questions · your answer locks when you choose it

1. What is $F(1, 2)$ for $F = \langle -y, x \rangle$?

- **A** $\langle 2, -1 \rangle$
- **B** $\langle -1, -2 \rangle$
- **C** $\langle 1, 2 \rangle$
- **D** $\langle -2, 1 \rangle$

2. The field $\langle x, y \rangle$ points:

- **A** toward the origin
- **B** straight up
- **C** away from the origin
- **D** around the origin

3. What is ∇f for $f = x^2 + y^2$?

- **A** $\langle x, y \rangle$
- **B** $\langle 2x, 2y \rangle$
- **C** $2x + 2y$
- **D** $\langle 2, 2 \rangle$

4. A conservative field is:

- **A** the gradient of some function
- **B** any constant field only
- **C** never a gradient
- **D** always zero

5. Which field describes a counterclockwise swirl?

- **A** $\langle x, y \rangle$
- **B** $\langle y, x \rangle$
- **C** $\langle 1, 0 \rangle$
- **D** $\langle -y, x \rangle$

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. A vector field assigns:

A a number to each point
B a surface
C a vector to each point
D a curve to each point

2. $\int_C F \cdot \mathrm{d}\mathbf{r}$ measures:

A area
B mass
C work along C
D flux

3. Divergence measures:

A rotation
B spreading out (sources and sinks)
C curvature
D length

4. Curl measures:

A speed
B rotation
C spreading
D mass

5. Flux measures:

A flow through a surface
B flow along a curve
C rotation
D speed



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Series E · Linear Algebra (Class E1: Linear Systems & Matrices)