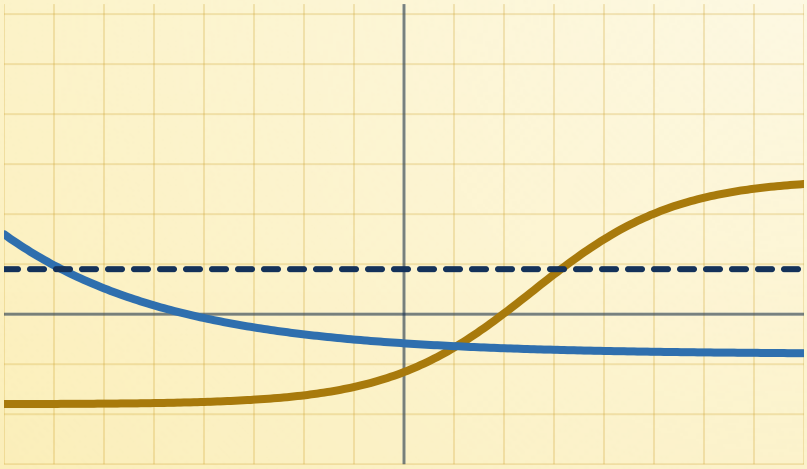
KECUDO

2026 EDITION · COLLEGE MATH SERIES



DIFFERENTIAL EQUATIONS

Class F1

First-Order ODEs

Equations that describe change

PREMIUM EDITION · KECUDO · F1

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

What Is a Differential Equation?

A **differential equation** (DE) relates an unknown function to its derivatives. An **ordinary** DE (ODE) has one independent variable. The **order** is the highest derivative present.

Equation	Order	Linear?
$y' = 2y$	1	yes
$y'' + 4y = \sin t$	2	yes
$y' = y^2 - t$	1	no (y^2)
$y\,y'' = 1$	2	no ($y\,y''$)

WORKED EXAMPLE **Check a solution**

1. Is $y = e^{2x}$ a solution of $y' = 2y$?
2. $y' = 2e^{2x} = 2y$. Yes.

✓ Substitute and compare both sides

WORKED EXAMPLE **Initial-value problem: $y' = 2y$, $y(0) = 3$**

1. All solutions: $y = Ce^{2x}$.
2. $y(0) = C = 3$.

✓ $y = 3e^{2x}$: the initial condition picks one curve from the family

GENERAL VS PARTICULAR

A first-order DE usually has a one-parameter family of solutions (the **general solution**). An initial condition picks out one **particular solution**.

●○○ FOUNDATION

Chapter 2

Direction Fields

For $y' = f(x, y)$, draw a short segment with slope $f(x, y)$ at many points. Solution curves follow the segments like iron filings around a magnet.

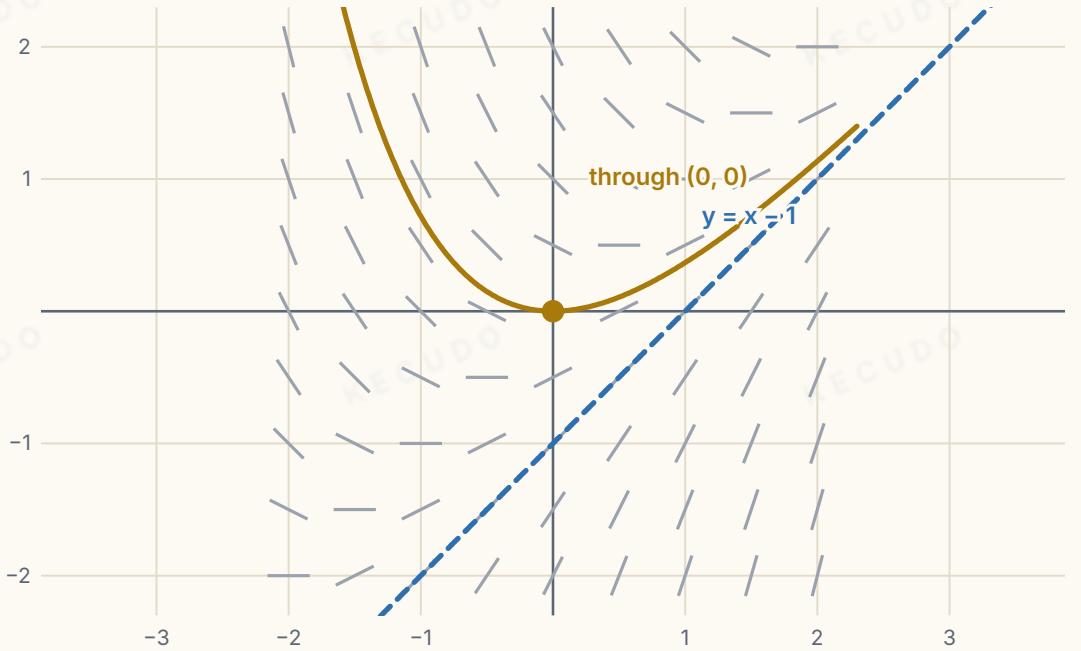


FIG 2.1 Direction field of $y' = x - y$. Every solution approaches the line $y = x - 1$.

WORKED EXAMPLE **Read the field**

1. Along $y = x$, slopes are 0 (an **isocline**).
2. The exact solutions are $y = x - 1 + Ce^{-x}$; the term Ce^{-x} fades, so every curve approaches $y = x - 1$.

✓ **You can predict long-run behavior without solving**

WHY DIRECTION FIELDS MATTER

Most nonlinear DEs can't be solved by formula. A direction field (or a computer's version of it) still shows how solutions behave.

CHECK YOUR UNDERSTANDING

Chapter 1: What Is a Differential Equation?

5 questions · your answer locks when you choose it

1. What is the order of $y''' + y' = x$?

- **A** 3
- **B** 1
- **C** 0
- **D** 2

2. Is $y' + y^2 = 0$ linear?

- **A** Only for $y > 0$
- **B** Yes
- **C** No
- **D** Can't tell

3. Does $y = e^{-x}$ solve $y' + y = 0$?

- **A** No
- **B** Only at $x = 0$
- **C** Yes
- **D** Only for $x > 0$

4. The general solution of $y' = 2y$ is:

- **A** e^{2x} only
- **B** $x^2 + C$
- **C** $2x + C$
- **D** Ce^{2x}

5. Solve $y' = 2y$, $y(0) = 5$.

- **A** $5e^{2x}$
- **B** $2e^{5x}$
- **C** $5x^2$
- **D** $e^{2x} + 5$

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. A first-order ODE involves:

A y'' | **B** integrals only | **C** y' but no higher derivatives | **D** no derivatives

2. A separable equation has the form:

A $y' = g(x) + h(y)$ | **B** $y' = y^2 + x$ | **C** $y' = g(x)h(y)$ | **D** $y'' = f$

3. The integrating factor for $y' + Py = Q$ is:

A $e^{\int Q\,dx}$ | **B** $e^{\int P\,dx}$ | **C** P | **D** $1/P$

4. An equilibrium of $y' = f(y)$ is:

A a maximum of f | **B** where $f' = 0$ | **C** $y = 0$ always | **D** a zero of f

5. Euler's method follows:

A secant lines | **B** exact solutions | **C** parabolas | **D** tangent lines in small steps



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class F2 · Second-Order Linear ODEs