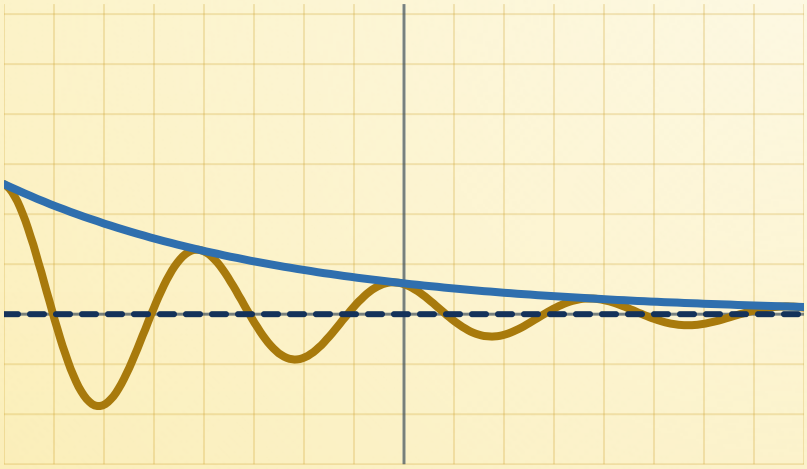
KECUDO

2026 EDITION · COLLEGE MATH SERIES



DIFFERENTIAL EQUATIONS

*Class F2*

# Second-Order Linear ODEs

*Springs, vibrations and resonance*

PREMIUM EDITION · KECUDO · F2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

*Chapter 1*

# Second-Order Basics

$$a(x)y'' + b(x)y' + c(x)y = g(x)$$

If $g = 0$ the equation is **homogeneous**. Newton's law $F = \mathrm{ma}$ makes second-order equations everywhere: position, velocity and acceleration all appear.

**SUPERPOSITION**

If $y_1$ and $y_2$ solve a linear homogeneous equation, so does $c_1\,y_1 + c_2\,y_2$. Two independent solutions give the general solution.

WORKED EXAMPLE **An initial-value problem needs two conditions**

1. $y'' = -9.8$ (a dropped part), $y(0) = 20$, $y'(0) = 0$.
2. Integrate twice: $y = 20 - 4.9t^2$.

✓ **Position and velocity at the start**

FOUNDATION

*Chapter 2*

# Homogeneous, Constant Coefficients

$$ay'' + by' + cy = 0\text{: try } y = e^{rx} \Rightarrow ar^2 + br + c = 0$$

The **characteristic equation** turns a differential equation into a quadratic.

**WORKED EXAMPLE** $y'' - 5y' + 6y = 0, y(0) = 1, y'(0) = 0$

1. $r^2 - 5r + 6 = 0 \to r = 2, 3$: $y = c_1 e^{2x} + c_2 e^{3x}$.
2. $c_1 + c_2 = 1$, $2c_1 + 3c_2 = 0 \to c_1 = 3$, $c_2 = -2$.

✓ $y = 3e^{2x} - 2e^{3x}$

**WHY EXPONENTIALS?**

Derivatives of $e^{rx}$ are multiples of $e^{rx}$, so each term is proportional to the same function and can cancel.

CHECK YOUR UNDERSTANDING

# Chapter 1: Second-Order Basics

5 questions · your answer locks when you choose it

**1.** A second-order IVP needs how many initial conditions?

- **A** 1
- **B** 2
- **C** 0
- **D** 3

**2.** Is $y'' + xy = 0$ linear?

- **A** Can't tell
- **B** No
- **C** Yes
- **D** Only for $x > 0$

**3.** Superposition holds for:

- **A** nonlinear equations
- **B** linear homogeneous equations
- **C** all equations
- **D** only first-order

**4.** $y'' = -9.8$, $y(0) = 20$, $y'(0) = 0$: when does $y = 0$?

- **A** ≈ 4.08 s
- **B** ≈ 2.02 s
- **C** ≈ 1 s
- **D** ≈ 20 s

**5.** The general solution of a 2nd-order linear homogeneous ODE has:

- **A** no constants
- **B** 3 constants
- **C** 1 constant
- **D** 2 constants

FINAL EXAM

# 25 Questions on the Whole Book

Parts A–E · 5 questions each

## Part A · Basics

**1.** The characteristic equation of $ay'' + by' + cy = 0$ is:

**A** $ar + b = 0$ | **B** $ar^2 + br + c = 0$

**C** $a + b + c = 0$ | **D** $r^2 = c$

**2.** Complex roots produce solutions that:

**A** are linear | **B** oscillate

**C** only grow | **D** are constant

**3.** The Wronskian tests:

**A** independence of solutions | **B** damping

**C** stability | **D** resonance

**4.** The general solution of a nonhomogeneous equation is:

**A** $y_c$ | **B** $y_c + y_p$

**C** $y_p$ | **D** $y_c - y_p$

**5.** Resonance occurs when:

**A** forcing matches the natural frequency | **B** $m$ is large

**C** $k = 0$ | **D** damping is large




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*Thank you for learning with Kecudo.*

NEXT IN THE SERIES

Class F3 · Laplace Transforms