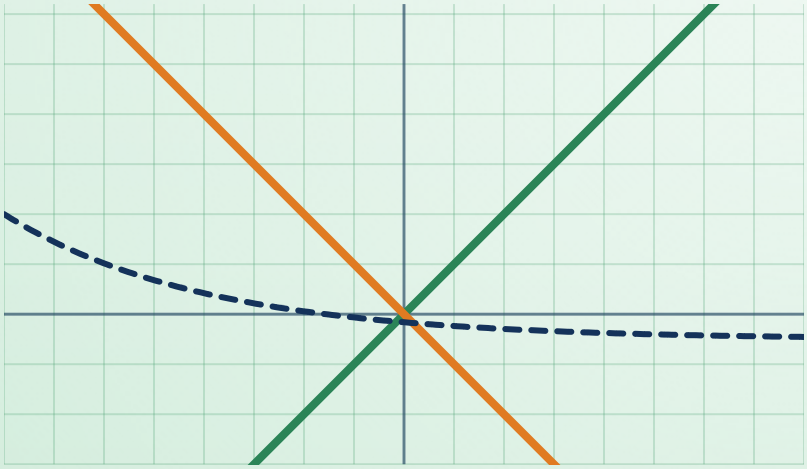
KECUDO

2026 EDITION · COLLEGE MATH SERIES



LINEAR ALGEBRA

Class E3

Determinants, Eigenvalues & Eigenvectors

The directions a matrix can't turn

PREMIUM EDITION · KECUDO · E3

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Determinants

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

For larger matrices, use **cofactor expansion** along any row or column: multiply each entry by its cofactor $C_{ij} = (-1)^{i+j} \det A_{ij}$ (with A_{ij} the matrix left after deleting row i and column j) and add.

WORKED EXAMPLE **A = [1 2 3; 0 1 4; 5 6 0], along row 1**

1. $1(0 - 24) - 2(0 - 20) + 3(0 - 5)$.
2. $= -24 + 40 - 15$.

✓ **$\det A = 1$**

TRIANGULAR SHORTCUT

The determinant of a triangular matrix is the product of its diagonal entries. $\det [2\ 7\ 1;\ 0\ 3\ 5;\ 0\ 0\ 4] = 2 \cdot 3 \cdot 4 = 24$.

Sign pattern for cofactors
+ − +
− + −
+ − +

WATCH OUT

Pick the row or column with the most zeros: every zero is a cofactor you don't need to compute.

FOUNDATION

Chapter 2

Properties of Determinants

Row operation	Effect on det
Add a multiple of one row to another	no change
Swap two rows	changes sign
Multiply a row by k	multiplies det by k

$$\det(AB) = \det A \cdot \det B,\ \det A^T = \det A,\ \det A^{-1} = 1/\det A$$

THE INVERTIBILITY TEST

A is invertible exactly when $\det A \neq 0$. This adds one more item to the Invertible Matrix Theorem.

WORKED EXAMPLE **Use row reduction**

1. $[1\ 2; 3\ 4] \to [1\ 2; 0\ -2]$ by $R_2 - 3R_1$ (no change).
2. Triangular: $1(-2)$.

✓ **det = −2**

WATCH OUT

$\det(A + B) \neq \det A + \det B$ in general, and $\det(kA) = k^n \det A$ for an $n \times n$ matrix.

CHECK YOUR UNDERSTANDING

Chapter 1: Determinants

5 questions · your answer locks when you choose it

1. What is $\det [3\ 1; 4\ 2]$?

A 10 | **B** 2

C 5 | **D** −2

2. What is $\det [2\ 0\ 0; 0\ 5\ 0; 0\ 0\ -1]$?

A 6 | **B** −10

C 10 | **D** 0

3. The cofactor sign at position (1, 2) is:

A + | **B** depends on the entry

C always 0 | **D** −

4. What is $\det I_4$?

A 1 | **B** 4

C 16 | **D** 0

5. The best row for cofactor expansion has:

A the largest numbers | **B** no zeros

C the most zeros | **D** negative numbers

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. $\det A \neq 0$ means A is:

A invertible | **B** symmetric

C singular | **D** zero

2. An eigenvector satisfies:

A $Av = v$ only | **B** $Av = 0$

C $\det v = \lambda$ | **D** $Av = \lambda v$, $v \neq 0$

3. Eigenvalues solve:

A $A - \lambda = 0$ | **B** $\det A = 0$

C $Av = b$ | **D** $\det(A - \lambda I) = 0$

4. A is diagonalizable when it has:

A nonzero determinant | **B** n independent eigenvectors

C a zero eigenvalue | **D** real entries

5. $|\det A|$ measures:

A the rank | **B** the angle

C the trace | **D** how A scales area or volume



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class E4 · Orthogonality, Least Squares & SVD