KECUDO

2026 EDITION · COLLEGE MATH SERIES



LINEAR ALGEBRA

Class E4

Orthogonality, Least Squares & SVD

Right angles, best fits and data

PREMIUM EDITION · KECUDO · E4

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Inner Product & Length

$$u \cdot v = u_1 v_1 + \ldots + u_n v_n,\ \|u\| = \sqrt{(u \cdot u)},\ \text{dist}(u, v) = \|u - v\|$$

WORKED EXAMPLE $u = (1, 2, 2)$, $v = (2, 0, -1)$

1. $u \cdot v = 2 + 0 - 2 = 0$: they're **orthogonal**.
2. $\|u\| = 3$; the unit vector is $u/3 = (1/3, 2/3, 2/3)$.

✓ **Perpendicular vectors have dot product 0**

ORTHOGONAL COMPLEMENT

$W^\perp$ is the set of all vectors orthogonal to every vector in W. For any matrix, $(\text{Row } A)^\perp = \text{Nul } A$ and $(\text{Col } A)^\perp = \text{Nul } A^T$.

Pythagorean Theorem: $u \perp v \iff \|u + v\|^2 = \|u\|^2 + \|v\|^2$

FOUNDATION

Chapter 2

Orthogonal Sets

A set is **orthogonal** if every pair is orthogonal; **orthonormal** if, in addition, every vector has length 1. Nonzero orthogonal vectors are automatically independent.

In an orthogonal basis: $y = c_1\,u_1 + \ldots + c_p\,u_p$ with $c_j = (y \cdot u_j)/(u_j \cdot u_j)$

WORKED EXAMPLE **Basis $u_1 = (1, 1, 0)$, $u_2 = (1, -1, 0)$, $u_3 = (0, 0, 1)$; $y = (3, 1, 4)$**

1. $c_1 = 4/2 = 2$, $c_2 = 2/2 = 1$, $c_3 = 4/1 = 4$.

✓ **$y = 2u_1 + u_2 + 4u_3$: no system to solve**

ORTHOGONAL MATRICES

A square U with orthonormal columns has $U^T U = I$, so $U^{-1} = U^T$. It preserves lengths and angles: rotations and reflections.

CHECK YOUR UNDERSTANDING

Chapter 1: Inner Product & Length

5 questions · your answer locks when you choose it

1. What is $\|(3, 4)\|$?

- **A** 25
- **B** 5
- **C** 1
- **D** 7

2. Are $(1, 2)$ and $(2, -1)$ orthogonal?

- **A** Only if normalized
- **B** Can't tell
- **C** No
- **D** Yes

3. What is the unit vector in the direction of $(0, 3, 4)$?

- **A** $(0, 0.6, 0.8)$
- **B** $(0, 1, 1)$
- **C** $(0, 0.3, 0.4)$
- **D** $(0, 3, 4)$

4. $(\text{Row } A)^{\perp}$ =

- **A** $\{0\}$ always
- **B** Row A
- **C** Col A
- **D** Nul A

5. What is the distance between $(1, 1)$ and $(4, 5)$?

- **A** 5
- **B** 3
- **C** 25
- **D** 7

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. Vectors are orthogonal when:

- **A** their sum is 0
- **B** their dot product is 0
- **C** they're parallel
- **D** their lengths match

2. The least-squares solution satisfies:

- **A** $\hat{x} = 0$
- **B** $A^T \hat{x} = b$
- **C** $A\hat{x} = b$
- **D** $A^T A \hat{x} = A^T b$

3. Gram–Schmidt turns a basis into:

- **A** a bigger basis
- **B** zeros
- **C** an orthogonal basis
- **D** eigenvectors

4. Symmetric matrices have:

- **A** complex eigenvalues
- **B** no eigenvectors
- **C** determinant 0
- **D** real eigenvalues and orthogonal eigenvectors

5. Singular values are:

- **A** $\sqrt{(\text{eigenvalues of } A^T A)}$
- **B** eigenvalues of A
- **C** always 1
- **D** entries of A



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Series F · Differential Equations (Class F1: First-Order ODEs)