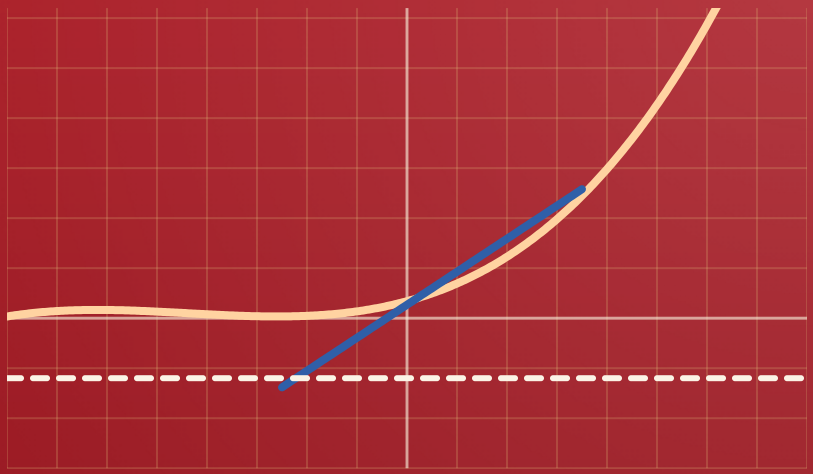
KECUDO

2026 EDITION · COLLEGE MATH SERIES



NUMERICAL METHODS

Class K1

Roots, Errors & Linear Systems

Computing answers you can trust

PREMIUM EDITION · KECUDO · K1

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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●○○ FOUNDATION

Chapter 1

Floating-Point Numbers

Computers store real numbers in **floating point**: $\pm(1.b_1b_2\ldots b_{52})_2 \times 2^e$ in IEEE double precision, about 16 significant decimal digits.

Quantity	Double precision
machine epsilon ε (gap after 1)	$2^{-52} \approx 2.22 \times 10^{-16}$
largest number	$\approx 1.8 \times 10^{308}$
smallest normal positive	$\approx 2.2 \times 10^{-308}$
0.1 + 0.2	0.30000000000000004

WORKED EXAMPLE **Why 0.1 + 0.2 ≠ 0.3**

1. $0.1 = 0.000110011001100\ldots_2$ repeats forever in binary, so it is rounded.
2. The two rounding errors don't cancel.

✓ **Never test floats with ==; use $|a - b| < \text{tolerance}$**

WORKED EXAMPLE **Kecudo's billing**

1. Adding 0.10 ten times in floating point gives 0.9999999999999999.
2. Kecudo stores money in integer cents instead.

✓ **Exact integers for money**

FOUNDATION

Chapter 2

Types of Error

Error	Source	Example
round-off	finite digits	0.1 stored inexactly
truncation	stopping an infinite process	$e^x \approx 1 + x + x^2/2$
data / model	inexact inputs or assumptions	a sensor reading ±0.5%

$$\text{absolute error} = |\tilde{x} - x| \quad \cdot \quad \text{relative error} = |\tilde{x} - x| \,/\, |x|$$

WORKED EXAMPLE **Catastrophic cancellation**

1. For $x = 10^8$, $\sqrt{(x^2 + 1)} - x$ computes as 0 (both terms agree to 16 digits).
2. Rewrite: $1/(\sqrt{(x^2 + 1)} + x) = 5 \times 10^{-9}$, correct.

✓ **Subtracting nearly equal numbers destroys digits**

RULE OF THUMB

Relative error measures digits of accuracy: relative error 10^{-6} means about 6 correct significant digits.

CHECK YOUR UNDERSTANDING

Chapter 1: Floating-Point Numbers

5 questions · your answer locks when you choose it

1. Machine epsilon in double precision ≈

- **A** 0.1
- **B** 10^{-308}
- **C** 10^{-8}
- **D** 2.2×10^{-16}

2. 0.1 + 0.2 == 0.3 in floating point is:

- **A** true
- **B** random
- **C** an error
- **D** false

3. Doubles carry about how many significant decimal digits?

- **A** 100
- **B** 16
- **C** 32
- **D** 8

4. Safer comparison of floats a and b:

- **A** a > b
- **B** round both to 0
- **C** |a − b| < tolerance
- **D** a == b

5. Money in software is best stored as:

- **A** integer cents
- **B** binary fractions
- **C** strings only
- **D** floats

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. Machine epsilon is:

- **A** zero
- **B** the gap between 1 and the next float
- **C** the smallest error possible in any computation
- **D** the largest float

2. Bisection needs:

- **A** f′
- **B** a matrix
- **C** a sign change on [a, b]
- **D** two close guesses

3. Newton's method converges with order:

- **A** 2
- **B** 3
- **C** 1
- **D** 1.618

4. κ(A) large means the system is:

- **A** easy
- **B** sparse
- **C** ill-conditioned
- **D** diagonal

5. Gauss–Seidel is an:

- **A** interpolation method
- **B** iterative method for linear systems
- **C** integration rule
- **D** root finder for f(x)

K

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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class K2 · Interpolation & Approximation