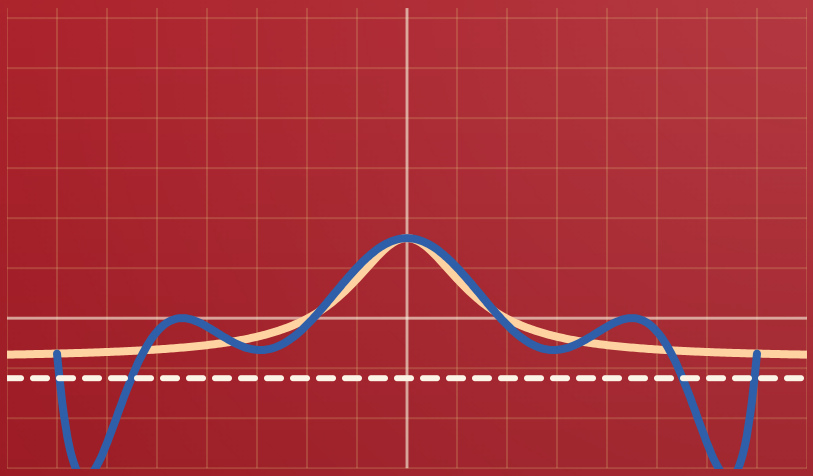
KECUDO

2026 EDITION · COLLEGE MATH SERIES



NUMERICAL METHODS

Class K2

Interpolation & Approximation

Fitting curves to data, wisely

PREMIUM EDITION · KECUDO · K2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Polynomial Interpolation

EXISTENCE AND UNIQUENESS

Through n + 1 points with distinct x-values there is exactly one polynomial of degree ≤ n.

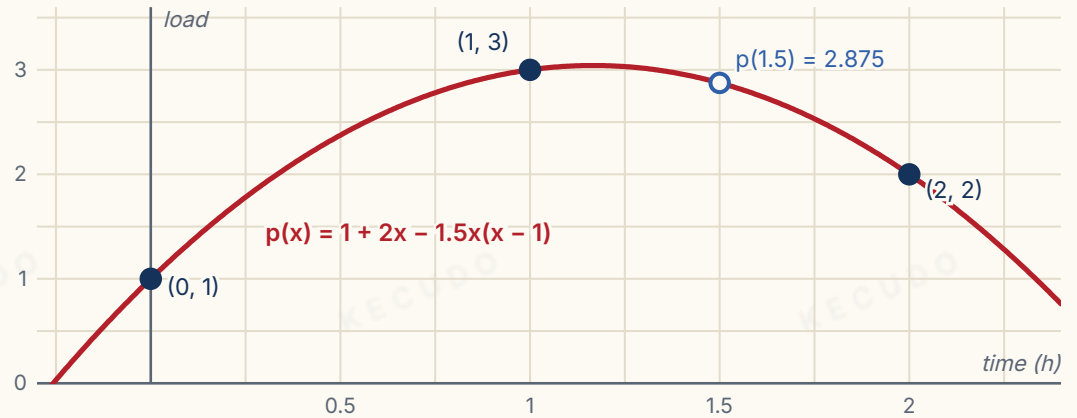


FIG 1.1 Kecudo's charger load at hours 0, 1, 2 (1, 3, 2 kW): the unique quadratic through the three points estimates 2.875 kW at 1.5 h.

WORKED EXAMPLE **The Vandermonde way**

1. Write $p(x) = a + bx + cx^2$, plug in the 3 points, solve 3 equations.
2. It works, but the Vandermonde matrix is ill-conditioned for many points: use Lagrange or Newton forms instead.

✓ **Same polynomial, better numerics**

FOUNDATION

Chapter 2

Lagrange Form

$$p(x) = \Sigma\, y_i\, L_i(x),\quad L_i(x) = \Pi_{j\neq i}\,(x - x_j)/(x_i - x_j)\quad (L_i = 1 \text{ at } x_i,\ 0 \text{ at the other nodes})$$

WORKED EXAMPLE **Points (0, 1), (1, 3), (2, 2)**

1. $L_0 = (x - 1)(x - 2)/2$, $L_1 = -x(x - 2)$, $L_2 = x(x - 1)/2$.
2. $p(1.5) = 1 \cdot (-0.125) + 3 \cdot 0.75 + 2 \cdot 0.375 = 2.875$.

✓ Matches Fig 1.1

PROS AND CONS

Lagrange form is easy to write down and analyze, but adding a new point means recomputing every L_i (the barycentric form fixes this).

CHECK YOUR UNDERSTANDING

Chapter 1: Polynomial Interpolation

5 questions · your answer locks when you choose it

1. Through 4 points (distinct x) there is a unique polynomial of degree at most:

- **A** 4
- **B** 2
- **C** 5
- **D** 3

2. Estimating between data points is:

- **A** regression
- **B** differentiation
- **C** interpolation
- **D** extrapolation

3. p(1.5) for the charger quadratic:

- **A** 3
- **B** 2.875
- **C** 2.5
- **D** 2

4. The Vandermonde system becomes:

- **A** exact
- **B** singular for 2 points
- **C** always diagonal
- **D** ill-conditioned for many points

5. Two points determine a unique:

- **A** cubic
- **B** circle
- **C** line
- **D** parabola

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. Interpolation passes the curve:

- **A** through every data point
- **B** through one point
- **C** near the points
- **D** through the mean

2. Runge's phenomenon is fixed by Chebyshev nodes or:

- **A** more equally spaced points
- **B** higher degree
- **C** Newton's method
- **D** splines

3. A cubic spline is continuous in:

- **A** nothing
- **B** f, f′, f″
- **C** f‴
- **D** f only

4. Least squares minimizes:

- **A** the sum of squared residuals
- **B** the number of knots
- **C** the degree
- **D** the max error

5. The FFT computes:

- **A** a discrete Fourier transform fast
- **B** integrals
- **C** splines
- **D** roots



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class K3 · Numerical Integration & ODE Solvers