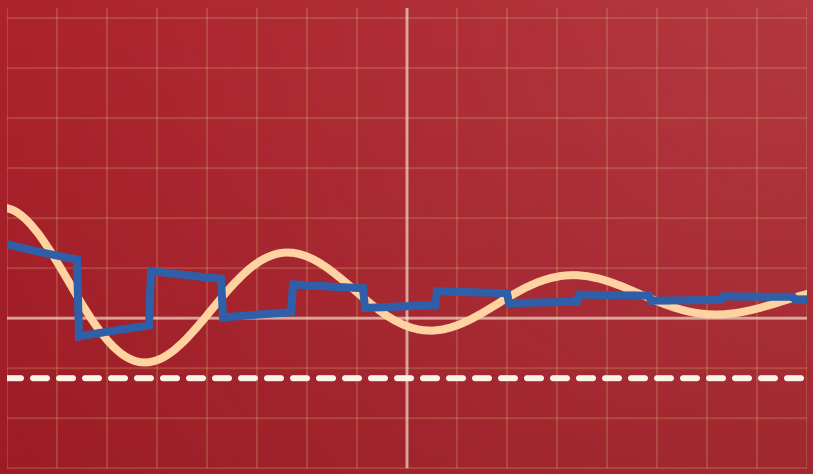
KECUDO

2026 EDITION · COLLEGE MATH SERIES



NUMERICAL METHODS

Class K3

Numerical Integration & ODE Solvers

Areas and motion, computed step by step

PREMIUM EDITION · KECUDO · K3

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

Contents

Part I · Foundations 4

1. Numerical Differentiation 5
2. Trapezoid Rule 7
3. Simpson's Rule 9
4. Error Analysis 11

Part I Review · 10 questions **13**

Part II · Core 15

5. Romberg Integration 16
6. Gaussian Quadrature 18
7. Adaptive Integration 20
8. Euler's Method 22
9. Runge–Kutta Methods 24

Part II Review · 10 questions **26**

Part III · Advanced 28

10. Multistep Methods 29
11. Stiff Equations 31
12. Solving Systems 33
13. Boundary-Value Problems (Shooting) 35

Part III Review · 10 questions **37**

Final Exam · 25 questions **39**

Your Results **44**

FOUNDATION

Chapter 1

Numerical Differentiation

forward: $f'(x) \approx (f(x + h) - f(x))/h$, error $O(h)$ · central: $f'(x) \approx (f(x + h) - f(x - h))/(2h)$, error $O(h^2)$

WORKED EXAMPLE **$f = e^x$ at $x = 1$, $h = 0.1$ (true 2.71828)**

1. Forward: $(e^{1.1} - e)/0.1 = 2.8588$ (error 0.14).
2. Central: $(e^{1.1} - e^{0.9})/0.2 = 2.7228$ (error 0.0045).

✓ **Central is far more accurate at the same h**

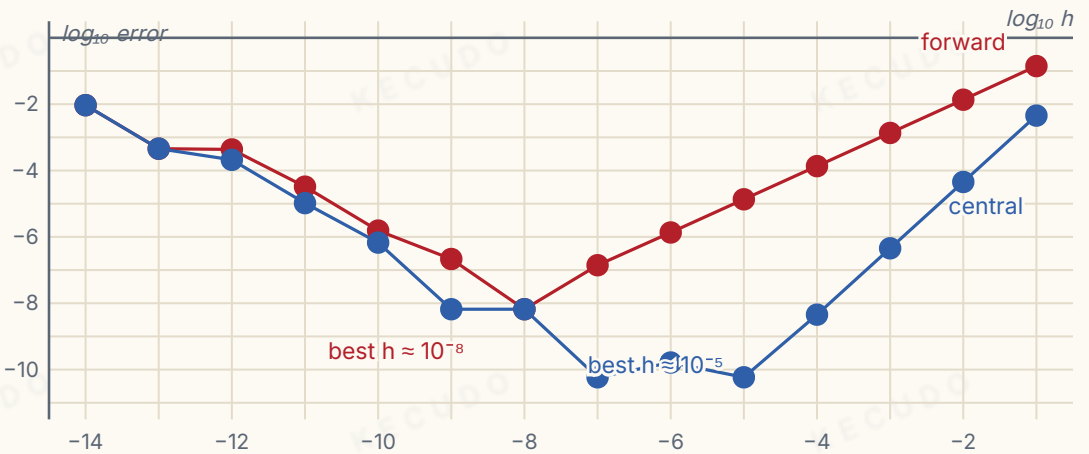


FIG 1.1 Error vs step size h. Shrinking h helps until round-off takes over: the best forward h is about 10^{-8}, the best central h about 10^{-5}.

THE ROUND-OFF TRADE-OFF

Truncation error shrinks with h, but dividing a tiny difference by a tiny h amplifies round-off. There is always a best h.

FOUNDATION

Chapter 2

Trapezoid Rule

$$\int_a^b f \approx h[f(x_0)/2 + f(x_1) + \cdots + f(x_{n-1}) + f(x_n)/2],\ h = (b - a)/n$$

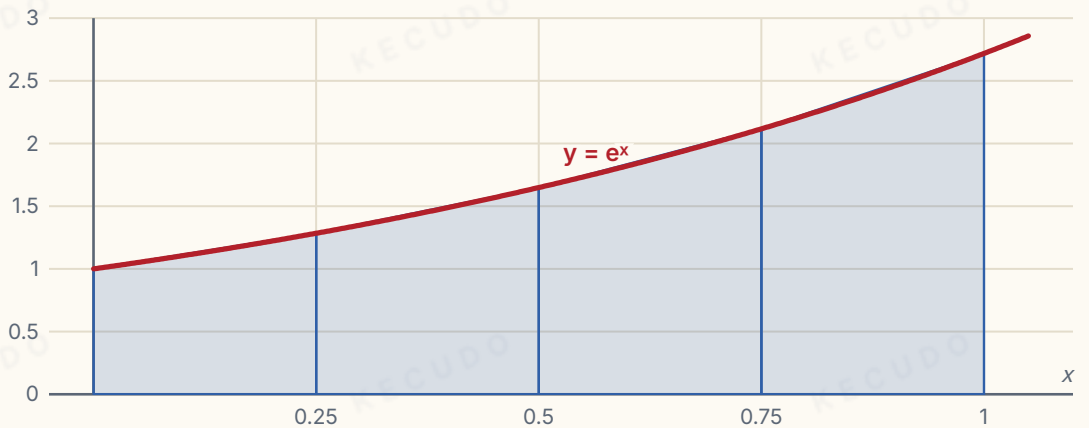


FIG 2.1 Four trapezoids under $y = e^x$ on [0, 1]. They slightly overestimate because e^x curves upward.

n	Trapezoid	Error (exact e − 1 = 1.718282)
1	1.859141	0.1409
2	1.753931	0.0356
4	1.727222	0.0089
8	1.720519	0.0022

WORKED EXAMPLE **Kecudo's GPS distance**

1. Speed (km/h) logged every minute; distance = ∫v dt.
2. Trapezoids on the 1-minute samples give the trip length to within a few metres.

✓ Integrating sampled data

CHECK YOUR UNDERSTANDING

Chapter 1: Numerical Differentiation

5 questions · your answer locks when you choose it

1. The central difference error is:

- **A** $O(h^2)$
- **B** $O(1)$
- **C** $O(h)$
- **D** $O(h^3)$

2. Forward difference for e^x at 1 with h = 0.1:

- **A** ≈ 3
- **B** ≈ 2.723
- **C** ≈ 2.859
- **D** 2.718

3. Making h extremely small (10^{-14}) gives:

- **A** a perfect derivative
- **B** large round-off error
- **C** an exact answer
- **D** zero error

4. Best h for forward differences in double precision:

- **A** ≈ 10^{-8}
- **B** ≈ 10^{-16}
- **C** ≈ 0.1
- **D** ≈ 1

5. Halving h in central differences cuts the truncation error by about:

- **A** 8
- **B** 16
- **C** 4
- **D** 2

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. Central differences have error order:

A 2 **B** 4
C 1 **D** 0

2. Simpson's rule is exact for polynomials up to degree:

A 5 **B** 1
C 3 **D** 2

3. Euler's method has global order:

A 2 **B** 3
C 4 **D** 1

4. RK4 has order:

A 1 **B** 4
C 2 **D** 5

5. Stiff problems call for:

A Fourier series **B** explicit Euler with large h
C bisection **D** implicit methods



KECUDO

Thank you for learning with Kecudo.

NEXT IN THE SERIES

Series L · Applied & Business Math