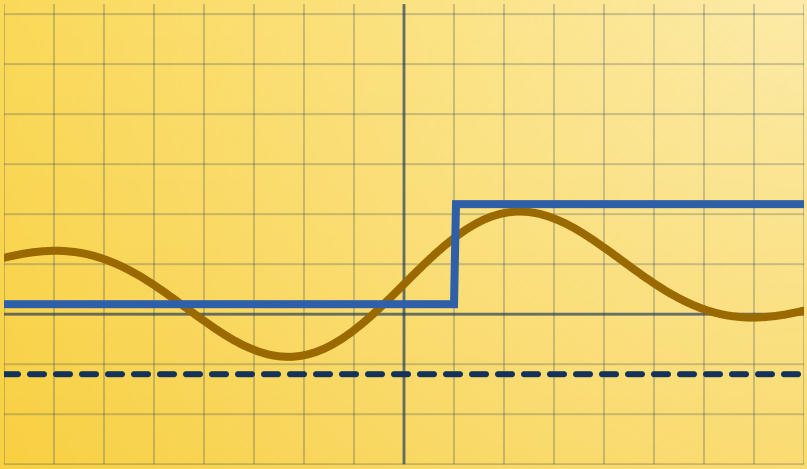
KECUDO

2026 EDITION · COLLEGE MATH SERIES



REAL ANALYSIS

Class I2

Continuity

No jumps, no gaps: the ε–δ way

PREMIUM EDITION · KECUDO ·

I2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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●○○ FOUNDATION

Chapter 1

Functional Limits

DEFINITION

$\lim_{x \to c} f(x) = L$ means: for every $\varepsilon > 0$ there is $\delta > 0$ such that $0 < |x - c| < \delta$ implies $|f(x) - L| < \varepsilon$.

WORKED EXAMPLE $\lim_{x \to 2} (3x + 1) = 7$

1. $|f(x) - 7| = |3x - 6| = 3|x - 2|$.
2. Choose $\delta = \varepsilon/3$. Then $|x - 2| < \delta$ gives $|f(x) - 7| < \varepsilon$.

✓ **Linear functions: $\delta = \varepsilon/|\text{slope}|$**

WORKED EXAMPLE $\lim_{x \to 3} x^2 = 9$

1. $|x^2 - 9| = |x - 3| \cdot |x + 3|$.
2. Restrict $\delta \le 1$, so $2 < x < 4$ and $|x + 3| < 7$.
3. Choose $\delta = \min(1, \varepsilon/7)$.

✓ **Bound the extra factor first, then pick δ**

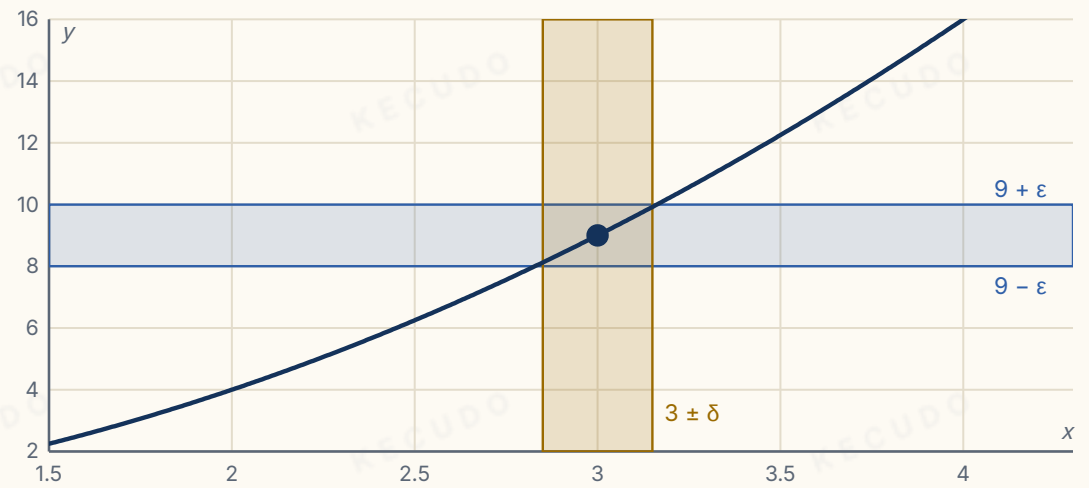


FIG 1.1 For $y = x^2$ near $x = 3$: every x in the δ-strip (gold) maps into the ε-strip around 9 (blue).

FOUNDATION

Chapter 2

The Sequential Criterion

SEQUENTIAL CRITERION

$\lim_{x \to c} f(x) = L$ if and only if $f(x_n) \to L$ for **every** sequence $x_n \to c$ with $x_n \neq c$.

WORKED EXAMPLE **sin(1/x) has no limit at 0**

1. $x_n = 1/(n\pi) \to 0$ gives $\sin(n\pi) = 0 \to 0$.
2. $y_n = 1/(2n\pi + \pi/2) \to 0$ gives $\sin = 1 \to 1$.

✓ **Two sequences, two different answers: no limit**

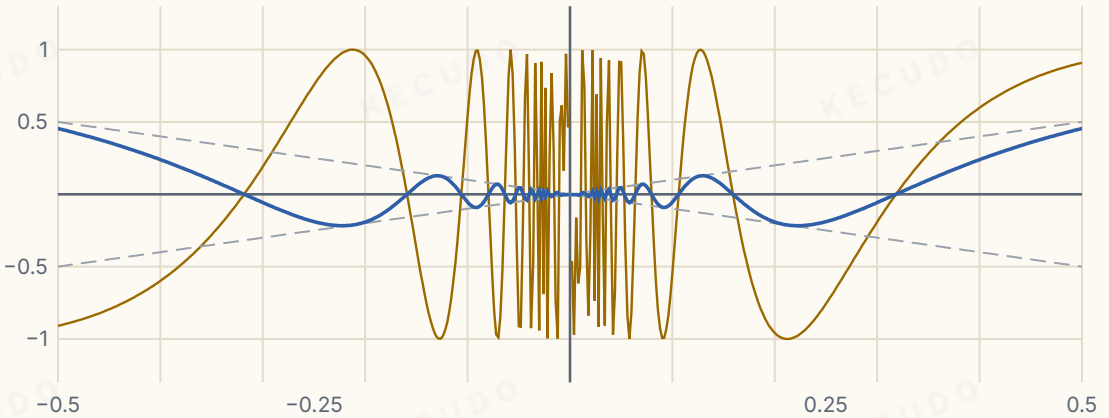


FIG 2.1 sin(1/x) (gold) oscillates forever near 0. x · sin(1/x) (blue) is squeezed between ±|x| (gray) and tends to 0.

USING IT

To prove a limit fails, find two sequences with different limits. To prove algebraic limit laws, borrow them from sequences.

CHECK YOUR UNDERSTANDING

Chapter 1: Functional Limits

5 questions · your answer locks when you choose it

1. In the ε–δ definition, δ may depend on:

- **A** x
- **B** ε (and c)
- **C** nothing
- **D** f(x)

2. For lim (5x − 2) at x = 1, a good δ is:

- **A** 5ε
- **B** ε/5
- **C** ε
- **D** ε − 5

3. For x^2 near 3 with $\delta \le 1$, $|x + 3| <$

- **A** 3
- **B** 6
- **C** 4
- **D** 7

4. The definition excludes x = c because:

- **A** the limit ignores f(c)
- **B** δ must be 0
- **C** it's a typo
- **D** f(c) is always 0

5. For ε = 0.07 in the x^2 example, δ = min(1, ε/7) =

- **A** 1
- **B** 0.49
- **C** 0.07
- **D** 0.01

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. Continuity at c means $\lim_{x \to c} f(x) =$

A ∞ **B** 0
C c **D** f(c)

2. The EVT needs:

A a monotone f **B** a continuous f on [a, b]
C an open interval **D** a differentiable f

3. Uniform continuity uses one δ for:

A all points **B** each ε separately at each x
C no ε **D** one point

4. Uniform convergence means:

A f_n is constant **B** sup $|f_n - f| \to 0$
C derivatives converge **D** $f_n(x) \to f(x)$ at one point

5. A jump discontinuity has one-sided limits that:

A don't exist **B** exist and differ
C are equal **D** are infinite



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class I3 · Differentiation