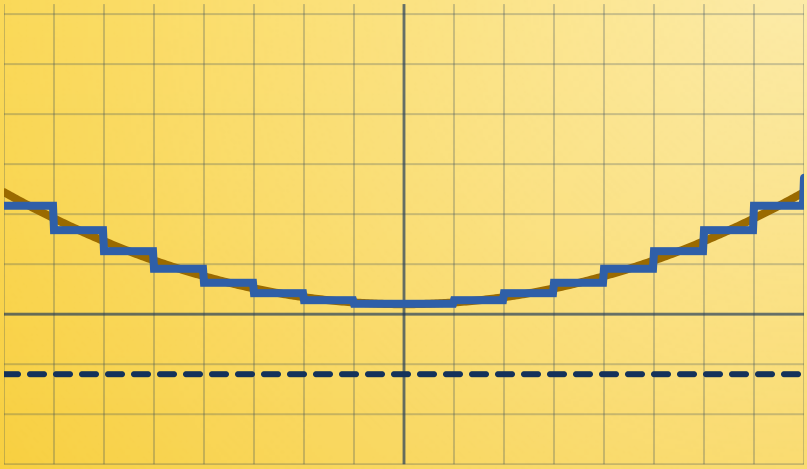
KECUDO

2026 EDITION · COLLEGE MATH SERIES



REAL ANALYSIS

Class I4

Riemann Integration

Area, made rigorous

PREMIUM EDITION · KECUDO ·

I4

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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●○○ FOUNDATION

Chapter 1

Partitions

A **partition** P of [a, b] is a finite list $a = x_0 < x_1 < \cdots < x_n = b$. Its **mesh** is the longest subinterval. A **refinement** $P' \supseteq P$ adds more points.

WORKED EXAMPLE **Uniform partition of [0, 1] into 4**

1. P = {0, 1/4, 1/2, 3/4, 1}; mesh 1/4.
2. Adding 1/8 gives a refinement with mesh still 1/4.

✓ **Refining never increases the mesh**

WHY PARTITIONS?

Instead of guessing areas, analysis squeezes the area between rectangle sums built on partitions and makes them meet.

Notation	Meaning
$\Delta x_k = x_k - x_{k-1}$	width of the k-th piece
$m_k = \inf f$ on $[x_{k-1}, x_k]$	lowest value on the piece
$M_k = \sup f$ on $[x_{k-1}, x_k]$	highest value on the piece

FOUNDATION

Chapter 2

Upper & Lower Sums

$$U(f, P) = \Sigma\, M_k\, \Delta x_k \;\cdot\; L(f, P) = \Sigma\, m_k\, \Delta x_k \;\cdot\; L(f, P) \le U(f, P)$$

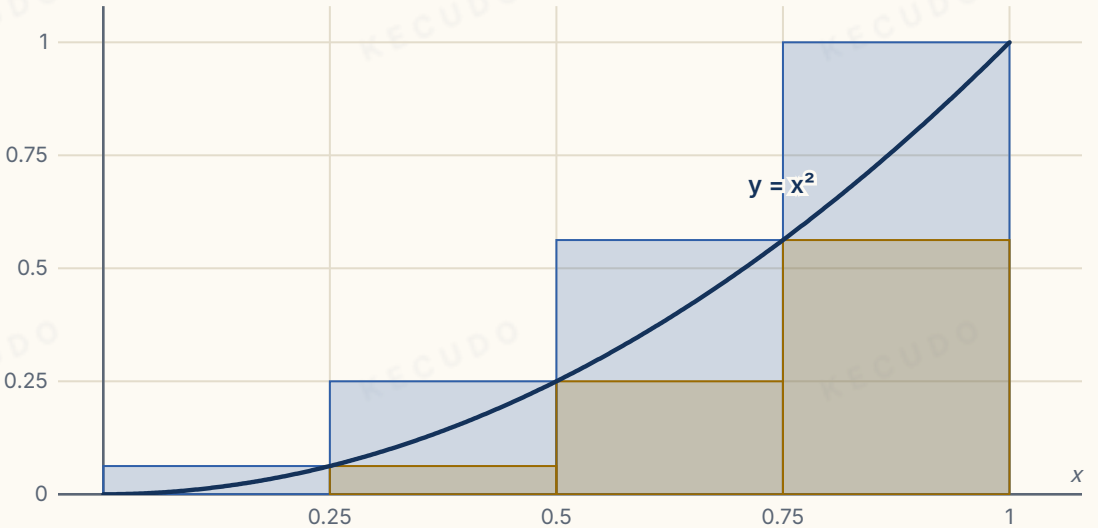


FIG 2.1 f(x) = x² on [0, 1] with 4 pieces. Upper rectangles (blue) overshoot; lower rectangles (gold) undershoot.

WORKED EXAMPLE **Compute for n = 4**

1. L = (0 + 1 + 4 + 9)/16 · 1/4 = 14/64 ≈ 0.219.
2. U = (1 + 4 + 9 + 16)/16 · 1/4 = 30/64 ≈ 0.469.
3. In general U − L = 1/n.

✓ The true area 1/3 is trapped between them

REFINEMENT LEMMA

Refining a partition can only raise L and lower U. So every lower sum is ≤ every upper sum.

CHECK YOUR UNDERSTANDING

Chapter 1: Partitions

5 questions · your answer locks when you choose it

1. A partition of [0, 2] into 4 equal parts has mesh:

A 2 **B** 4
C 1/4 **D** 1/2

2. Adding points to a partition gives a:

A coarsening **B** refinement
C subset **D** sequence

3. M_k is the:

A width **B** midpoint
C sup of f on the k-th piece **D** inf

4. Δx_3 for $P = \{0, 1, 3, 6\}$ is:

A 1 **B** 3
C 6 **D** 2

5. A partition must be:

A infinite **B** random
C uniform **D** finite

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. U(f, P) uses the:

A sup of f on each piece
B midpoint
C left endpoint
D inf

2. f is Riemann integrable when:

A $f \geq 0$
B f is differentiable
C upper and lower integrals are equal
D f is continuous

3. FTC 1: $F(x) = \int_a^x f$ has $F'(c) = f(c)$ where:

A never
B f is integrable only
C f is continuous at c
D always

4. A set of measure zero can be covered by intervals of total length:

A less than any $\varepsilon > 0$
B 0 exactly with one interval
C ∞
D exactly 1

5. Lebesgue's criterion: integrable $\Leftrightarrow$ bounded and discontinuities of:

A measure zero
B zero count
C positive measure
D finite number



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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Series J · Abstract Algebra